Name: SOLUTIONS
Date: _____

Midyear Review - 02
Functions

1. Functions (Linear)

$y = ax + b$

Abby, Breanne and Cate decide to join the same gym.
The gym charges a ***one-time membership fee*** as well as a ***small fee each time*** someone visits.

x_1 y_1 x = visits, y = $.

- **Abby visits the gym 64 times and pays $ 400**
- **Breanne pays $ 425 to visit the gym 84 times**

y_2 x_2

If Cate visits the gym 120 times, how much should she expect to pay? $x = 120$

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{425 - 400}{84 - 64} = \frac{25}{20} = 1.25$$

$y = ax + b$
$y = 1.25x + b$
$400 = 1.25(64) + b$
$400 = 80 + b$
$-80 \quad -80$
$320 = b$

$y = 1.25x + 320$
$y = 1.25(120) + 320$
$y = 150 + 320$
$y = 470$

Answer: Cate would expect to pay: $ 470

2. Functions (Quadratic)

What is the rule for a **quadratic function** that passes through the **origin** and point **(5, -3.125)?**

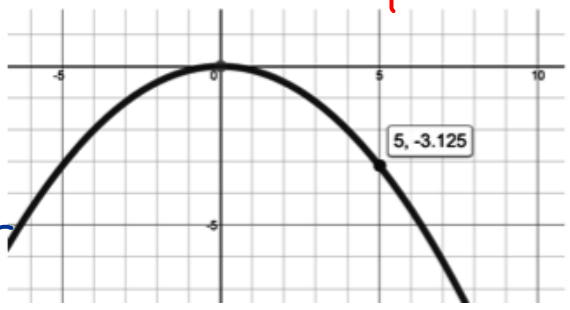
$y = ax^2$ formula.

$-3.125 = a(5)^2$

$\frac{-3.125}{25} = \frac{a(25)}{25}$

$-0.125 = a$

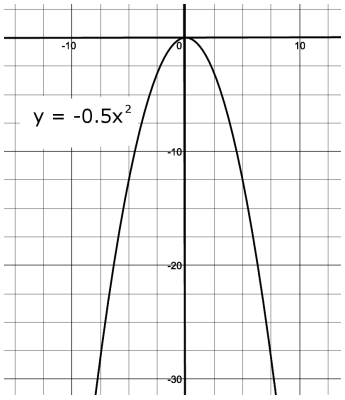
x y



'a' is pointing down.
'a' must be negative.

Answer: $y =$ $-0.125x^2$

3. Functions (Quadratic)



$y = -0.5x^2$

If $\mathbf{y = -112.5}$, what is the value of 'x'?

$y = a x^2$

$y = -0.5 x^2$

$\frac{-112.5}{-0.5} = \frac{-0.5x^2}{-0.5}$

$\sqrt{225} = \sqrt{x^2}$

$15 = x$

Answer: (15 , -112.5) .

4. Functions (Exponential)

A viral infection **doubles** in size every **4 hours**.
If the infection starts with **6 viral agents**, how many would there be at the end of **2 full days**?

c = 2

a = 6

48 hrs.

$x = \frac{48 \text{ hours}}{4 \text{ hours}} = 12$ times.

$y = a \cdot c^x$

$y = 6 \cdot (2)^{12}$

$y = 24\,576$ viral agents.

Answer: There would be 24 576 viral agents at the end of 2 full days

5. Functions (Exponential)

goes down.

The value of a car **declines** at an average rate of **8.5 % per year**.
The car was purchased in **2009** for **$ 35 000.**
How much is the car worth in **2016?**

$c = 100\% - 8.5\%$

$c = 91.5\%$ (divide by 100)

$c = 0.915$

$y = a \cdot c^x$

$y = 35000 \cdot 0.915^7$

$y = 18\,793.85$

$a = 35\,000$ (starting number)

$x = 2016 - 2009 = 7$ years

Answer: The car will be worth $ 18 793.85 in 2016.

6. Functions (Exponential)

C is increasing (growing)

Luke decided to go to the bank and invest some money.
In **2004**, he deposited the money in an account with an annual **average rate of return of 25%**
In **2007**, the value of the account was **$ 2 343.75**. — must be 'y'

How much money did Luke invest, initially? — find 'a'.

C = 100% + 25%.

C = 125 %.

C = 1.25

X = 2007 − 2004

X = 3 years

$y = a \cdot c^x$

$2343.75 = a\,(1.25)^3$

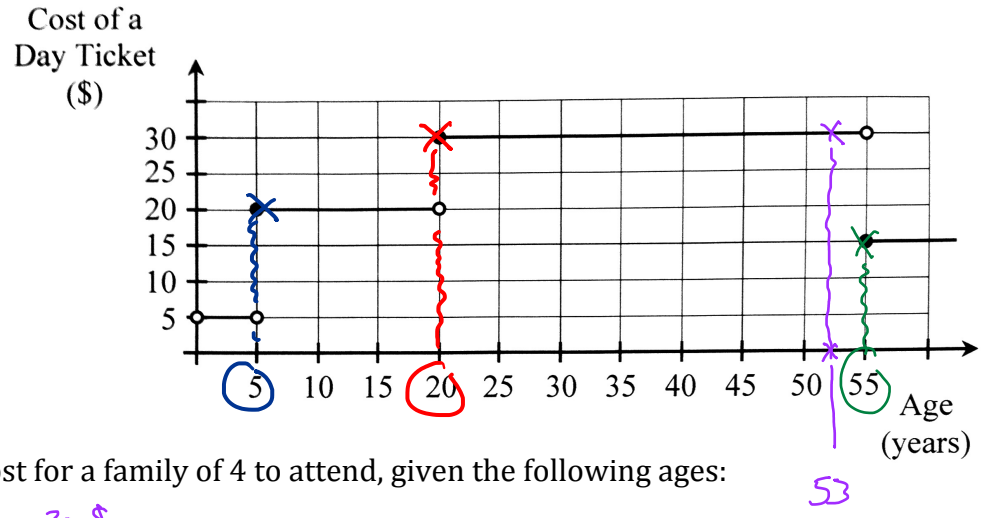
$\frac{2343.75}{1.953125} = \frac{a\,(1.953125)}{1.953125}$

$1200 = a$

Answer: Luke's initial investment was for $ 1200.00

7. Functions (Step)

It is family day at the local rink and ticket prices vary depending on age.



53

How much would it cost for a family of 4 to attend, given the following ages:

- **Mom**: age 53 → 30 $
- **Dad**: age 55 → 15 $
- **Kid #1**: age 20 → 30 $
- **Kid # 2**: age 5 → 20 $

95 $ total

Answer: It would cost $ 95.00

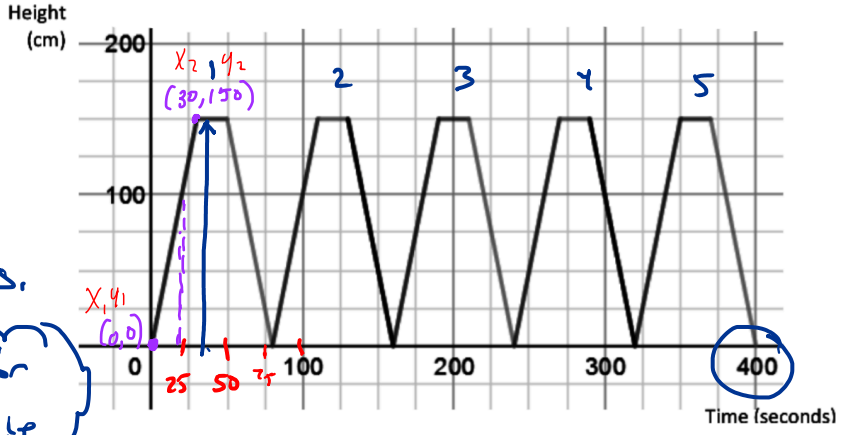
8. Functions (Periodic)

The graph below is a periodic function that shows the height of a ball off the ground as a function of time.

- The ball rises and falls at the same speed.
- When the ball reaches a max height of 150 cm, it stays put for 20 seconds before falling again.

How high above the ground is the ball after 15 minutes?

Time.



Periodic Function:

Period: 5 full cycles in 400 seconds.

$= \frac{400}{5} =$ 80 seconds for 1 full cycle.

TIME UNTIL WE STOP: 15 min × 60 sec/min = 900 seconds

$900 \div 80 = 11.25$ → $0.25 \times 80 =$ 20 seconds into a new cycle is where we stop

equation where 20 seconds hits the function:

$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{150 - 0}{30 - 0} = \frac{150}{30} = 5$

$y = 5x + 0$

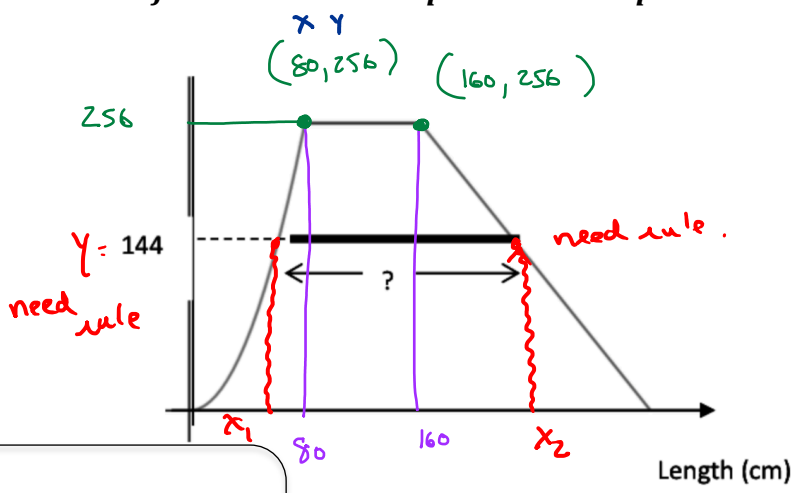
$y = 5(20) + 0$

$y = 100$

Answer: After 15 minutes, the ball will be 100 cm off the ground.

9. ***Functions (Piecewise)***

The graph below represents the outline of a skateboard ramp which corresponds to a piecewise function defined by:



$$f(x) = \begin{cases} ax^2 & \text{if } 0 \le x \le 80 \\ 256 & \text{if } 80 \le x \le 160 \\ -1.25x + b & \text{if } 160 \le x \le \ ? \end{cases}$$

You must use this information to answer the problem

For security purposes, a strip of reflective tape will be placed on the ramp at a height of **144 cm.**

What is the length of this piece of reflective tape? (Length along the x-axis**)**

$y = ax^2$

$256 = a(80)^2$

$\frac{256}{6400} = \frac{a(6400)}{6400}$

$0.04 = a$

$y = 0.04x^2$

$\frac{144}{0.04} = \frac{0.04x^2}{0.04}$

$\sqrt{3600} = \sqrt{x^2}$

$60 = x$

$y = -1.25x + b$

$256 = -1.25(160) + b$

$256 = -200 + b$

$+200 \quad +200$

$456 = b$

$y = -1.25x + 456$

$144 = -1.25x + 456$

$-456 \quad -456$

$\frac{-312}{-1.25} = \frac{-1.25x}{-1.25}$

$249.6 = x$

Answer: The length of the tape will be 189.6 cm.

$\text{Diff} = x_2 - x_1 = 249.6 - 60 = 189.6$